

NUMERICAL SIMULATION OF MAGNETOHYDRODYNAMICS MIXED CONVECTION FLOW WHEN THE MAGNETIC FIELD, FREE STREAM VELOCITY AND SURFACE TEMPERATURE OSCILLATE SIMULTANEOUSLY

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ABSTRACT

This paper tries to apply the finite difference method along with perturbation technique to solve the nonlinear two dimensional unsteady coupled equations. To remove the difficulties in solving the coupled equations primitive variable formulation (PVF) for finite difference method and stream function formulation (SFF) for perturbation technique is proposed. The physical phenomena describe the hydromagnetic mixed convection flow when the magnetic field, free stream velocity and surface temperature oscillate in magnitude simultaneously. For this purpose different values of mixed convection parameter λ , Prandtl number, Pr, the magnetic Prandtl number Pm , and the magnetic force parameter S are discussed in terms of amplitudes and phases angle of shear stress, rate of heat transfer and current density. The effects of these parameters on the amplitude of oscillation of the transient shear stress, rate of heat and current density are also discussed.

Keywords: Fluctuating flow, Magnetohydrodynamic, Mixed Convection, Magnetized Plate, Current Density

1. INTRODUCTION

Lighthill [1] obtained the solutions for unsteady forced convection flow past a plate and circular cylinder with small amplitude oscillation. Ackerberg studied the boundary layer flow on a semi infinite plate due to small fluctuations in the magnitude of free stream velocity. [2]-[13] discussed the convective boundary layer flow along a vertical plate when the plate is subject to transverse mechanical vibration analytically as well as numerically. Steady MHD forced and free convection flow past a vertical plate has been studied widely because of its importance in aeronautics, missile aerodynamics and in some other engineering applications. With all these wide range of applications [14]-[23] studied the MHD boundary layer uniform flow past a magnetized and non-magnetized plate for different values of magnetic force parameter S and magnetic Prandtl number Pm . The two dimensional steady boundary layer flow and heat transfer of electrically conducting, viscous, incompressible fluid past a semi-infinite plate in the presence of uniform magnetic field has been investigated by [24]-[25]. Ingham [26] studied the boundary layer flow of electrically conducting gas with an aligned magnetic field on a semi-infinite flat plate at large distances from the plate placed at zero incidence.

Unsteady forced convection boundary layer flow through saturated porous medium has been carried out by Hossain and Banu [27]. Later, this

model has been extended Hossain et al [28] to an unsteady free convection flow of viscous incompressible and electrically conducting fluid along a vertical plate in the presence of a variable

transverse magnetic field when the surface temperature of the plate oscillates with small amplitude. The natural convection boundary layer flow of viscous incompressible fluid along a vertical plate has been discussed by Roy and Hossain [30]. Effect of small amplitude oscillation in the wall temperature on the natural convection flow from a cylinder of elliptic cross section has been investigated by Jaman and Hossain [31].

The hydromagnetic mixed convection flow of viscous incompressible fluid past a magnetized vertical porous plate has been discussed by Ashraf and Hossain [32]. Recently, Ashraf et al. [33] studied the effect of thermal radiation-conduction on hydromagnetic mixed

convection flow of viscous incompressible fluid past a magnetized plate. The above literature survey proposed that the unsteady hydromagnetic mixed convection flow past a magnetized vertical heated plate, when the magnetic field, free stream velocity and surface temperature oscillates in magnitude have not yet been studied. In view of above literature survey, we propose the study of unsteady hydromagnetic mixed convection flow past a magnetized surface and highlights the effects of varying the mixed convection parameter λ , the Prandtl number, Pr , the magnetic Prandtl number Pm , and the magnetic force parameter S in terms of amplitudes and phases angle of shear stress, rate of heat transfer and current density.

2. BASIC EQUATIONS AND FLOW CONFIGURATION

We consider a unsteady two-dimensional magnetohydrodynamic mixed convection flow of an electrically conducting, viscous and incompressible fluid past a heated and magnetized vertical plate by including radiation effects in the energy equation. The flow configuration and the co-ordinate system is shown in Fig. 1. We have taken x -axis along the surface and y -axis is normal to it. In this figure δ_M , δ_T and δ_H represent momentum, thermal and magnetic boundary layer thicknesses. Further more we assume that the surface temperature, magnetic field and the free stream are oscillating with time about constant means. The momentum, magnetic, and energy flow field with the influence of radiation effects are now governed by the following dimensionless equations.

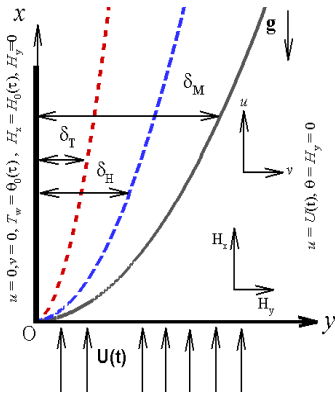


Fig 1. The coordinate system and flow configuration

which describe the unsteady hydromagnetic mixed convection flow past a vertical plate. The boundary layer equations are

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \quad (1)$$

$$\frac{\partial \bar{u}}{\partial \tau} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} = \frac{dU}{d\tau} + \frac{\partial^2 \bar{u}}{\partial y^2}$$

$$+ S(\bar{H}_x \frac{\partial \bar{H}_x}{\partial x} + \bar{H}_y \frac{\partial \bar{H}_x}{\partial y}) + \lambda \bar{\theta}$$

(2)

$$\frac{\partial \bar{H}_x}{\partial x} + \frac{\partial \bar{H}_y}{\partial y} = 0$$

(3)

$$\frac{\partial \bar{H}_x}{\partial \tau} + \bar{u} \frac{\partial \bar{H}_x}{\partial x} + \bar{v} \frac{\partial \bar{H}_x}{\partial y} - \bar{H}_x \frac{\partial \bar{u}}{\partial x} - \bar{H}_y \frac{\partial \bar{u}}{\partial y} = \frac{1}{Pm} \frac{\partial^2 \bar{H}_x}{\partial y^2}$$

(4)

$$\frac{\partial \bar{T}}{\partial \tau} + \bar{u} \frac{\partial \bar{T}}{\partial x} + \bar{v} \frac{\partial \bar{T}}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \bar{T}}{\partial y^2}$$

(5)

The boundary conditions are taken as form $\bar{u}(\bar{x}, 0) = 0, \bar{v}(\bar{x}, 0) = 0, H_x(\bar{x}, 0, \tau) = H_0(\tau)$

$$\bar{T}(\bar{x}, 0, \tau) = T_0(\tau), \bar{u}(\bar{x}, \infty, \tau) = U(\tau),$$

$$\bar{H}_x(\bar{x}, \infty, \tau) = 0, \bar{T}(\bar{x}, \infty, \tau) = 0$$

(6)

Here $H_0(\tau)$, $\theta_0(\tau)$ are magnetic field intensity and surface temperature oscillation which are given as below:

$$U(\tau) = 1 + \varepsilon e^{i\tau}, H_0(\tau) = 1 + \varepsilon e^{i\tau}, \theta_0(\tau) = 1 + \varepsilon e^{i\tau}$$

(7)

In above expression it is assumed that ε is 1 amplitude of oscillation of free stream velocity, surface, magnetic intensity and surface temperature which is very small than unity.

Knowing the values of the dependent variables from (1)-(5), one can obtain the values of values of shear stress τ_m , current density J_m and the rate of heat transfer q_m from the following relations.

$$\tau_m = \left(\frac{\partial \bar{u}}{\partial y} \right)_{\bar{y}=0}, J_m = \left(\frac{\partial \bar{H}_x}{\partial y} \right)_{\bar{y}=0}$$

$$J_m = \left(\frac{\partial \bar{H}_x}{\partial y} \right)_{\bar{y}=0}, q_m = - \left(\frac{\partial \bar{T}}{\partial y} \right)_{\bar{y}=0} \quad (8)$$

Since ε the amplitude of oscillation of free stream velocity, surface, magnetic intensity and surface temperature is small one can consider the solution of the equations (1)-(5) together with boundary conditions (6) to be of the following form in terms of steady and fluctuating part of the flow variables \bar{u} , \bar{v} and \bar{H}_x , \bar{H}_y, T as the sum of steady and fluctuating components.

$$\bar{u} = [U_0(x, y) + \varepsilon e^{i\tau} U_1(x, y)], \bar{v} = \xi^{\frac{1}{2}} [V_0(x, y) + \varepsilon e^{i\tau} V_1(x, y)]$$

$$\bar{H}_x = [\bar{H}_{x_0}(x, y) + \varepsilon e^{ix} \bar{H}_{x_1}(x, y)], \bar{H}_y = \xi^{-1/2} [U_0(x, y) + \varepsilon e^{ix} U_1(x, y)]$$

$$\bar{T} = [\theta_0(x, y) + \varepsilon e^{ix} \theta_1(x, y)], Y = \xi^{-1/2} y, \quad \xi = x$$

where, $U_0, V_0, H_{x_0}, H_{y_0}, \theta_0$ and $U_1, V_1, H_{x_1}, H_{y_1}, \theta_1$ are respectively, the real and fluctuating parts of the flow variables.

Steady Parts

To find the solutions of the equations given in (1)-(5), we now introduce the expressions given above for \bar{u}, \bar{v} and \bar{H}_x, \bar{H}_y, T following equations for the steady mean parts of the flow are obtained

$$\xi \frac{\partial U_0}{\partial \xi} - \frac{Y}{2} \frac{\partial U_0}{\partial Y} + \frac{\partial V_0}{\partial Y} = 0 \quad (9)$$

$$\xi U_0 \frac{\partial U_0}{\partial \xi} + \left(V_0 - \frac{1}{2} Y U_0 \right) \frac{\partial U_0}{\partial Y} = \frac{\partial^2 U_0}{\partial Y^2}$$

$$+ S \left[\left(\bar{H}_{y_0} - \frac{Y}{2} \bar{H}_{x_0} \right) \frac{\partial \bar{H}_{x_0}}{\partial Y} + \xi \frac{\partial \bar{H}_{x_0}}{\partial \xi} \right] + \lambda \theta_0 \quad (10)$$

$$\xi \frac{\partial \bar{H}_{x_0}}{\partial \xi} - \frac{Y}{2} \frac{\partial \bar{H}_{x_0}}{\partial Y} + \frac{\partial \bar{H}_{y_0}}{\partial Y} = 0 \quad (11)$$

$$\xi U_0 \frac{\partial \bar{H}_{x_0}}{\partial \xi} + \left(V_0 - \frac{1}{2} Y U_0 \right) \frac{\partial \bar{H}_{x_0}}{\partial Y} - \xi \bar{H}_{x_0} \frac{\partial U_0}{\partial \xi}$$

$$- \left(V_0 - \frac{1}{2} \bar{H}_{x_0} \right) \frac{\partial U_0}{\partial Y} = \frac{1}{Pm} \frac{\partial^2 \bar{H}_{x_0}}{\partial Y^2} \quad (12)$$

$$\xi U_0 \frac{\partial \theta_0}{\partial \xi} + \left(V_0 - \frac{1}{2} Y U_0 \right) \frac{\partial \theta_0}{\partial Y} = \frac{1}{Pr} \frac{\partial^2 \theta_0}{\partial Y^2} \quad (13)$$

The boundary conditions to be satisfied by the above system of equations are

$$U_0(\xi, 0) = V_0(\xi, 0) = 0, \quad \bar{H}_{y_0}(\xi, 0) = 0, \quad \bar{H}_{x_0}(\xi, 0) = 1,$$

$$\theta_0(\xi, 0) = 1, U_0(\xi, \infty) = 1, \quad \bar{H}_{x_0}(\xi, \infty) = 0, \quad \theta_0(\xi, \infty) = 0 \quad (14)$$

Solutions of equations (9)-(14) are obtained using finite difference method together with Gaussian elimination technique. From this we also have

$$V_0(i, j) = V_0(i-1, j) - \frac{1}{2} \left(\xi \frac{\Delta Y}{\Delta \xi} - Y_j \right) U_0(i, j)$$

$$+ \frac{1}{2} \xi_i \frac{\Delta Y}{\Delta \xi} U_0(i, j-1) - \frac{1}{2} Y_j U_0(i-1, j) \quad (15)$$

$$\bar{H}_{y_0}(i, j) = \bar{H}_{y_0}(i-1, j) - \frac{1}{2} \left(\xi \frac{\Delta Y}{\Delta \xi} - Y_j \right) \bar{H}_{x_0}(i, j)$$

$$+ \frac{1}{2} \xi_i \frac{\Delta Y}{\Delta \xi} \bar{H}_{x_0}(i, j-1) - \frac{1}{2} Y_j \bar{H}_{x_0}(i-1, j) \quad (16)$$

The values obtained by the functions

$U_0, V_0, \bar{H}_{x_0}, \bar{H}_{y_0}, \theta_0$ are then used in oscillating part of the problem obtained in the following section.

Oscillating Parts

In a similar manner discussed above the equations for the oscillating flow may be obtained as

$$\xi \frac{\partial U_1}{\partial \xi} - \frac{Y}{2} \frac{\partial U_1}{\partial Y} + \frac{\partial V_1}{\partial Y} = 0 \quad (17)$$

$$i \xi U_1 + \xi U_0 \frac{\partial U_1}{\partial \xi} + \left(V_0 - \frac{Y}{2} U_0 \right) \frac{\partial U_1}{\partial Y} + \left(V_1 - \frac{Y}{2} U_1 \right) \frac{\partial U_0}{\partial Y}$$

$$= i \xi + \frac{\partial^2 U_1}{\partial Y^2} + S \left[\xi \bar{H}_{x_0} \frac{\partial \bar{H}_{x_1}}{\partial \xi} + \left(\bar{H}_{y_0} - \frac{Y}{2} \bar{H}_{x_0} \right) \frac{\partial \bar{H}_{x_1}}{\partial Y} \right.$$

$$\left. + \left(\bar{H}_{y_1} - \frac{Y}{2} \bar{H}_{x_1} \right) \frac{\partial \bar{H}_{x_0}}{\partial Y} \right] + \lambda \bar{\theta}_1 \quad (18)$$

$$\xi \frac{\partial \bar{H}_{x_1}}{\partial \xi} - \frac{Y}{2} \frac{\partial \bar{H}_{x_1}}{\partial Y} + \frac{\partial \bar{H}_{y_1}}{\partial Y} = 0 \quad (19)$$

$$i \xi \bar{H}_{x_1} + \xi U_0 \frac{\partial \bar{H}_{x_1}}{\partial \xi} + \left(V_0 - \frac{Y}{2} U_0 \right) \frac{\partial \bar{H}_{x_1}}{\partial Y} + \left(V_1 - \frac{Y}{2} U_1 \right) \frac{\partial \bar{H}_{x_0}}{\partial Y} - i \bar{H}_{x_0} \frac{\partial U_1}{\partial \xi}$$

$$+ \left(\bar{H}_{y_0} - \frac{Y}{2} \bar{H}_{x_0} \right) \frac{\partial U_1}{\partial Y} + \left(\bar{H}_{y_1} - \frac{Y}{2} \bar{H}_{x_1} \right) \frac{\partial U_0}{\partial Y} = \frac{1}{Pm} \frac{\partial^2 \bar{H}_{x_1}}{\partial Y^2} \quad (20)$$

$$i \xi \bar{\theta}_1 + \xi U_0 \frac{\partial \bar{\theta}_1}{\partial \xi} + \left(V_0 - \frac{Y}{2} U_0 \right) \frac{\partial \bar{\theta}_1}{\partial Y} + \left(V_1 - \frac{Y}{2} U_1 \right) \frac{\partial \bar{\theta}_0}{\partial Y} = \frac{1}{Pr} \frac{\partial^2 \bar{\theta}_1}{\partial Y^2} \quad (21)$$

The boundary conditions are as follows:

$$U_1(\xi, 0) = V_1(\xi, 0) = 0, H_{x_1}(\xi, 0) = 1, \bar{H}_{y_1}(\xi, 0) = 0,$$

$$\bar{\theta}_1(\xi, 0) = 0, U_1(\xi, \infty) = 1, \bar{H}_{x_1}(\xi, \infty) = 0, \bar{\theta}_1(\xi, \infty) = 0 \quad (22)$$

Solutions of equations (17)-(21) satisfying the conditions given in (22) are obtained employing the finite difference method for all ξ that discussed in the preceding section for the steady part of the problem.

Now we find the values of the physical quantities, like, the skin friction τ , rate of heat transfer χ and the current density H at the surface of plate.

Here, the expressions for amplitudes and phases angles of the skin friction, rate of heat transfer and current density are given as below:

$$A_s = \sqrt{\tau_r^2 + \tau_i^2}, A_\chi = \sqrt{\chi_r^2 + \chi_i^2}, A_m = \sqrt{H_r^2 + H_i^2}$$

$$\phi_s = \arctan\left(\frac{\tau_i}{\tau_r}\right), \phi_\chi = \arctan\left(\frac{\chi_i}{\chi_r}\right), \phi_m = \arctan\left(\frac{H_i}{H_r}\right) \quad (23)$$

where $(\tau_r, \tau_i), (\chi_r, \chi_i)$ and (H_r, H_i) are corresponding real and imaginary parts of the coefficients of skin friction, rate of heat transfer and current density at the surface. Recently this method has been used successfully by Roy and Hossain [31], Jaman and Hossain [32].

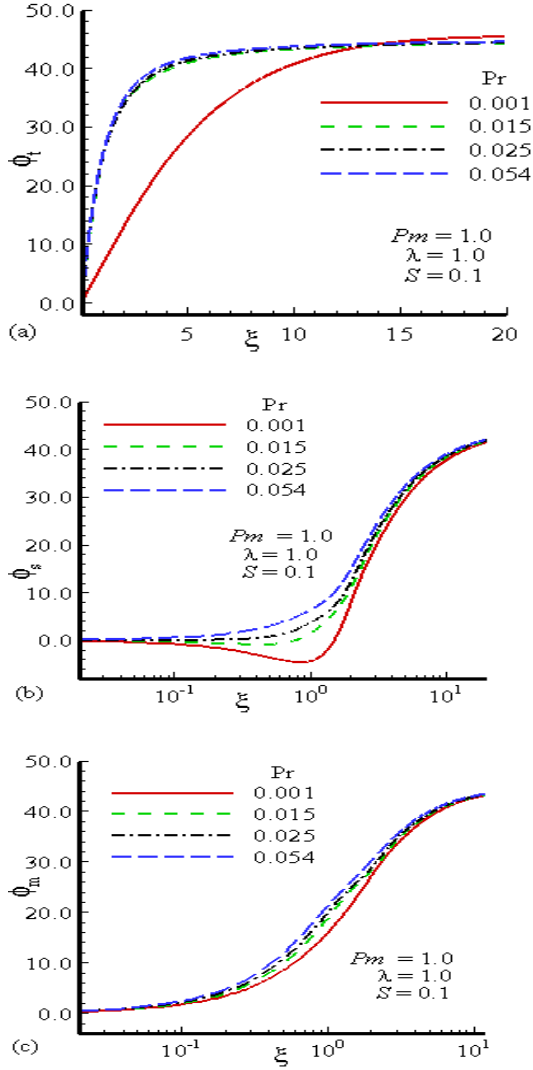


Fig 2. Numerical solution of phase angle of heat transfer, skin friction and current density for different values $Pr = 0.001, 0.015, 0.025, 0.054$ while $\lambda = 1.0, Pm = 1.0$ and $S = 0.1$

Further solutions are obtained for small and large values of ξ . Solutions for small ξ are obtained using the regular perturbation method. Finally matched asymptotic solutions for large ξ also obtained using the appropriate scaling factors. Details of the solutions are excluded here to economize the space of the journal. However, asymptotic solution thus obtained for large ξ are presented here are in terms of local skin friction, current density and rate of heat transfer as given below:

$$\begin{aligned} \tau &= s\xi^{\frac{1}{2}} + \xi^{-\frac{1}{2}} \frac{\lambda}{s(1-Pr)} - \frac{5}{4} i\xi^{-1} (c_0 + Sd_0\sqrt{Pm}) \\ H &= -sPm\xi - \frac{\xi^{-1}}{2} \left[(c_0\sqrt{Pm} + d_0)s + (c_0\sqrt{Pm} + d_0)s + \frac{9}{2} (c_0 + d_0)\sqrt{Pm} \right] \\ \chi &= -s\xi^{\frac{1}{2}}\sqrt{Pr} - \xi^{-1}\sqrt{Pr} \end{aligned} \quad (24)$$

where $s = (i+1)/\sqrt{2}$ and c_0, d_0 are known values

from steady solution.
0.1.

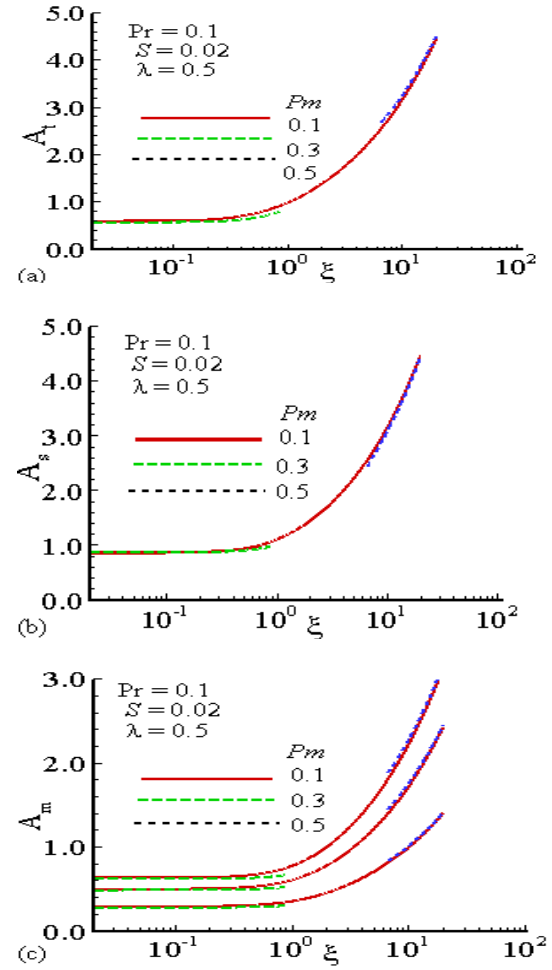


Fig 3. Amplitude of heat transfer, skin friction and current density for different values $Pm=0.1, 0.3, 0.5$, while $\lambda = 0.5$ and $S=0.02, Pr=0.1$.

By separating real and imaginary part from equation (24), we can find the numerical values amplitudes and phases angle of coefficients of skin friction, heat transfer and current density for large ξ from the following relations by following Hossain and Banu [27].

$$\begin{aligned} A_s &= \sqrt{\tau_r''^2 + \tau_i''^2}, \quad A_r = \sqrt{\chi_r'^2 + \chi_i'^2}, \quad A_m = \sqrt{H_r''^2 + H_i''^2} \\ \phi_s &= \arctan\left(\frac{\tau_i''}{\tau_r''}\right), \quad \phi_r = \arctan\left(\frac{\chi_i'}{\chi_r'}\right), \quad \phi_m = \arctan\left(\frac{H_i''}{H_r''}\right) \end{aligned}$$

The solution obtain by these relations for large ξ is given in Figures 2 and 3.

2. RESULTS AND DISCUSSIONS

In the present problem magnetohydrodynamics mixed convection flow past a magnetized vertical plate when magnetic field, free stream velocity and surface temperature oscillate has been investigated numerically. For numerical solution of the dimensionless equation

that govern the flow we have employed the finite difference approach with Gaussian elimination technique for entire value of ξ and asymptotic series solution for small and large value of ξ .

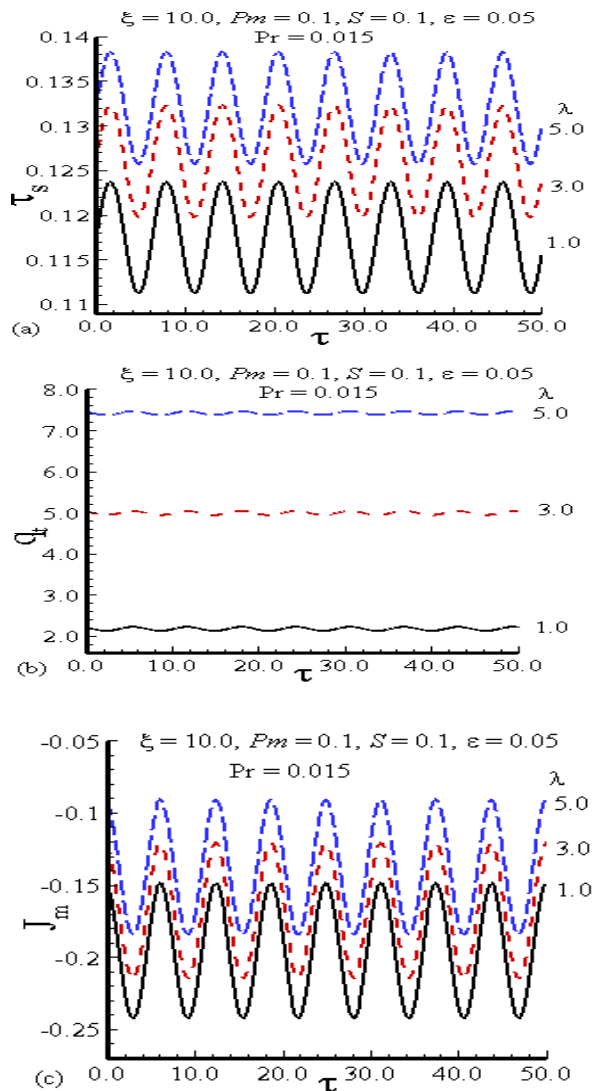


Fig 4. Transient skin friction rate of heat transfer, and current density for different values $\lambda = 1.0, 3.0, 5.0$ while $\xi = 10.0$ and $S=0.1, Pm=0.1, Pr=0.015$ and $\epsilon = 0.05$

The rate of heat transfer, skin friction and current density in terms of amplitude is exhibited in Figures 2(a)-2(c). It is observed that with the increase of Prandtl number Pr the amplitude of rate of heat transfer, skin friction and current density decreases. The variation in magnetic Prandtl number Pm against ξ on the amplitude of rate of heat transfer, skin friction and current density keeping other parameters constant is depicted in Figures 3(a)-3(c) by two methods finite difference method and perturbation technique. It is interesting to observe that the both methods are within good agreement.

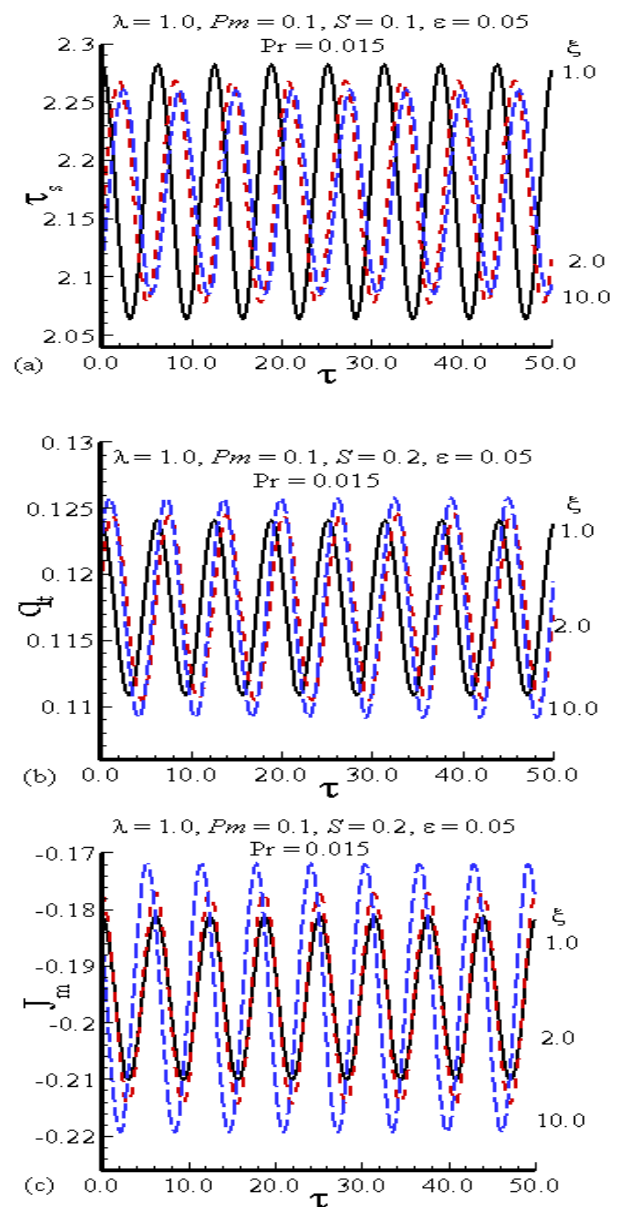


Fig 5. Transient skin friction rate of heat transfer, and current density for different values $\xi = 1.0, 2.0, 10.0$, while $\lambda = 1.0$ and $S=0.21, Pm=0.1, Pr=0.015$ and $\epsilon = 0.05$

Figures 4(a)-4(c) displays the transient skin friction, rate of heat transfer and current density for different values of mixed convection parameter λ . From these figures it is noted that the transient skin friction, rate of heat transfer and current density increases with the increase of mixed convection parameter λ . The effects of different values of dimensionless parameter λ on transient skin friction, rate of heat transfer and current density are given in Figures 5(a)-5(c). It is observed that with the increase of dimensionless parameter ξ the transient skin friction decreases and transient rate of heat transfer and current density increases.

4. CONCLUDING REMARKS

A numerical simulation is performed to analyze the

results of some physical quantities those are very important to explain physical phenomena in heat transfer and boundary layer theory. Based on the results and discussion the following conclusions have been drawn.

It is concluded that with the increase of Prandtl number Pr the phase angle of rate of heat transfer, skin friction and current density decreases. The very poor role of magnetic Prandtl number Pm for the case of skin friction and heat transfer in terms of amplitude is seen but this is very prominent in the case of current density.

It is also noted that the transient skin friction, rate of heat transfer and current density increases with the increase of mixed convection parameter λ . The increase in the dimensionless parameter ξ the transient skin friction decreases and transient rate of heat transfer and current density increases. In this investigation the results are compared by two methods finite difference method and perturbation technique, it is seen that the results obtained by both methods are within good agreement.

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